basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12/*GRAAD* 12

MATHEMATICS P1/*WISKUNDE V1*

NOVEMBER 2015

MEMORANDUM

MARKS: 150
PUNTE: 150

This memorandum consists of 25 pages.
Hierdie memorandum bestaan uit 25 bladsye.

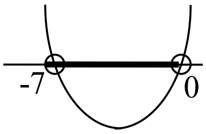
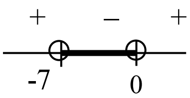
NOTE:
- If a candidate answers a question TWICE, only mark the FIRST attempt.
- Consistent accuracy applies in ALL aspects of the marking memorandum.

LET WEL:
- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord, merk slegs die EERSTE poging.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die memorandum van toepassing.*

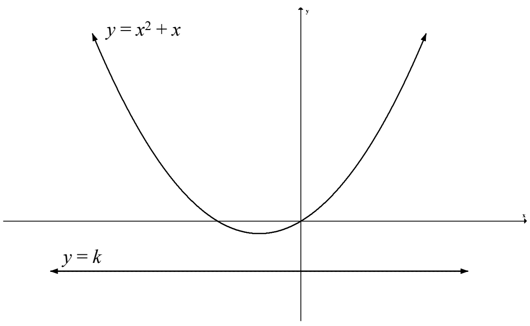
QUESTION/*VRAAG* 1

1.1.1	$x^2 - 9x + 20 = 0$ $(x-4)(x-5) = 0$ $x = 4$ or $x = 5$	✓factors ✓ $x = 4$ ✓ $x = 5$ (3)
1.1.2	$3x^2 + 5x - 4 = 0$ $x = \frac{-5 \pm \sqrt{(5)^2 - 4(3)(-4)}}{2(3)}$ $x = \frac{-5 \pm \sqrt{73}}{6}$ $x = -2{,}26$ or $x = 0{,}59$ **OR/*OF*** $x^2 + \frac{5}{3}x + \frac{25}{36} = \frac{4}{3} + \frac{25}{36}$ $\left(x + \frac{5}{6}\right)^2 = \frac{73}{36}$ $x + \frac{5}{6} = \pm\frac{\sqrt{73}}{6}$ $x = \frac{-5 \pm \sqrt{73}}{6}$ $x = -2{,}26$ or $x = 0{,}59$	✓standard form ✓substitution into correct formula ✓✓answers (4) ✓for adding $\frac{25}{36}$ on both sides ✓ $x = \frac{-5 \pm \sqrt{73}}{6}$ ✓✓answers (4)
1.1.3	$2x^{\frac{-5}{3}} = 64$ $x^{\frac{-5}{3}} = 32$ $x = \left(2^5\right)^{\frac{-3}{5}}$ $x = 2^{-3}$ or $\frac{1}{8}$ or $0{,}125$ **OR/*OF***	✓dividing both sides by 2 ✓$32 = 2^5$ or $64 = 2^6$ ✓raising RHS to $\frac{-3}{5}$ ✓answer (4)

	$2x^{\frac{-5}{3}}=64$ $x^{\frac{-5}{3}}=32$ $x=(32)^{\frac{-3}{5}}$ $x=\sqrt[5]{32^{-3}}$ $x=2^{-3}$ or $\frac{1}{8}$ or 0,125	✓dividing both sides by 2 ✓raising RHS to $\frac{-3}{5}$ ✓$\sqrt[5]{32^{-3}}$ ✓answer (4)
	OR/*OF* $\left(2x^{\frac{-5}{3}}\right)^{\frac{-3}{5}}=64^{\frac{-3}{5}}$ $0{,}659x=0{,}0825$ $x=0{,}125$	✓raising both sides to $\frac{-3}{5}$ ✓0,659 and 0,0825 ✓dividing both sides by 0,659 ✓answer (4)
	OR/*OF* $x^{\frac{-5}{3}}=32$ $\frac{-5}{3}\log x=\log 32$ $\log x=\frac{3}{-5}\log 32$ $\log x=-0{,}903$ $x=10^{-0{,}903}$ $=0{,}125$ or $\frac{1}{8}$	✓dividing both sides by 2 ✓logs on both sides ✓ $\log x=-0{,}903$ ✓answer (4)
1.1.4	$\sqrt{2-x}=x-2$ $2-x=(x-2)^2$ $2-x=x^2-4x+4$ $x^2-3x+2=0$ $(x-1)(x-2)=0$ $x=1$ or $x=2$ if $x=1$, $\sqrt{2-x}=1$ and $x-2=-1$ $x=2$ only	✓squaring both sides ✓factors ✓ $x=1$ or $x=2$ ✓ $x=2$ only (4)
	OR/*OF*	

	$\sqrt{2-x} = x-2$ $2-x = (x-2)^2$ $2-x = (2-x)^2$ $2-x=1$ or $2-x=0$ $x=1$ or $x=2$ if $x=1$, $\sqrt{2-x}=1$ and $x-2=-1$ $\therefore x=2$ only **OR/*OF*** $\sqrt{2-x} = x-2$ $2-x \geq 0$ and $x-2 \geq 0$ $x \leq 2$ and $x \geq 2$ $\therefore x=2$ only	✓squaring both sides ✓ $2-x=1$ or $2-x=0$ ✓ $x=1$ or $x=2$ ✓ $x=2$ only (4) ✓ $2-x \geq 0$ ✓ $x-2 \geq 0$ ✓ $x \leq 2$ and $x \geq 2$ ✓ $x=2$ (4)
1.1.5	$x^2+7x<0$ $x(x+7)<0$  **OR/*OF***  $-7<x<0$ **OR/*OF*** $x \in (-7;0)$	✓factors ✓✓inequality or interval (3)

1.2	The square of any number is always positive or zero So for the sum of two squares to be zero, both squares must be zero, i.e. *Die kwadraat van enige getal is altyd positief of nul. Vir die som van twee kwadrate om nul te wees, moet beide die kwadrate nul wees, d.i.* $(3x-y)^2=0$ and/*en* $(x-5)^2=0$ $3x-y=0$ and/*en* $x-5=0$ $x=5$ $3(5)-y=0$ $y=15$	✓ $3x-y=0$ ✓ $x-5=0$ ✓ $x=5$ ✓ $y=15$ (4)

1.3	$x^2+x=k$ $x^2+x-k=0$ $\Delta<0$ $b^2-4ac<0$ $1^2-4(1)(-k)<0$ $1+4k<0$ $k<\frac{-1}{4}$	✓standard form ✓ $\Delta<0$ ✓ $1^2-4(1)(-k)$ ✓ $k<\frac{-1}{4}$ (4)
	OR/*OF*	
	$x^2+x=k$ $x^2+x+\frac{1}{4}=k+\frac{1}{4}$ $\left(x+\frac{1}{2}\right)^2=k+\frac{1}{4}$ for nonreal roots $k+\frac{1}{4}<0$ $k<\frac{-1}{4}$	✓adds $\frac{1}{4}$ to both sides ✓ $\left(x+\frac{1}{2}\right)^2=k+\frac{1}{4}$ ✓ $k+\frac{1}{4}<0$ ✓ $k<\frac{-1}{4}$ (4)
	OR/*OF*	
	Consider the functions $y=x^2+x$ and $y=k$ *Beskou die funksies* $y=x^2+x$ *en* $y=k$ 	✓sketch or explanation
	Turning point of/*Draaipunt van* $y=x^2+x$ is $\left(\frac{-1}{2};\frac{-1}{4}\right)$ $x^2+x=k$ does not have real roots when the line $y=k$ does not intersect $y=x^2+x$. *$x^2+x=k$ het geen reële wortels as die lyn $y=k$ nie met $y=x^2+x$ sny nie.* Therefore $k<\frac{-1}{4}$	✓ $x=-\frac{1}{2}$ ✓ $y=-\frac{1}{4}$ ✓ $k<\frac{-1}{4}$ (4) **[26]**

QUESTION/*VRAAG* 2

2.1	$r=\dfrac{T_2}{T_1}$ $=\dfrac{5}{10}$ $=\dfrac{1}{2}$ $T_5=1{,}25\left(\dfrac{1}{2}\right)$ $=\dfrac{5}{8}$ or $0{,}625$ **OR/*OF*** $T_5=10\left(\dfrac{1}{2}\right)^4$ $=\dfrac{5}{8}$ or $0{,}625$	✓ $r=\dfrac{1}{2}$ ✓answer (2)
2.2	$T_n=10\left(\dfrac{1}{2}\right)^{n-1}$	✓substitutes $a=10$ into GP formula ✓substitutes $r=\dfrac{1}{2}$ into GP formula (2)
2.3	For convergence/*Om te konvergeer* $-1<r<1$ Since/*Aangesien* $r=\dfrac{1}{2}$ and/*en* $-1<\dfrac{1}{2}<1$ the sequence converges/*die ry konvergeer*	✓ $-1<r<1$ ✓ show that $r=\dfrac{1}{2}$ is $-1<r<1$ (2)
2.4	$S_\infty - S_n=\dfrac{a}{1-r}-\dfrac{a(1-r^n)}{1-r}$ $=\dfrac{10}{1-\frac{1}{2}}-\dfrac{10\left(1-\frac{1}{2}^n\right)}{1-\frac{1}{2}}$ $=20-20\left(1-\frac{1}{2}^n\right)$ $=20-20+20\left(\dfrac{1}{2}\right)^n$ $=20\left(\dfrac{1}{2}\right)^n$ **OR/*OF***	✓ $\dfrac{10}{1-\frac{1}{2}}$ ✓ $\dfrac{10\left(1-\frac{1}{2}^n\right)}{1-\frac{1}{2}}$ ✓ $20\left(1-\frac{1}{2}^n\right)$ ✓answer (4) ✓constructing the series

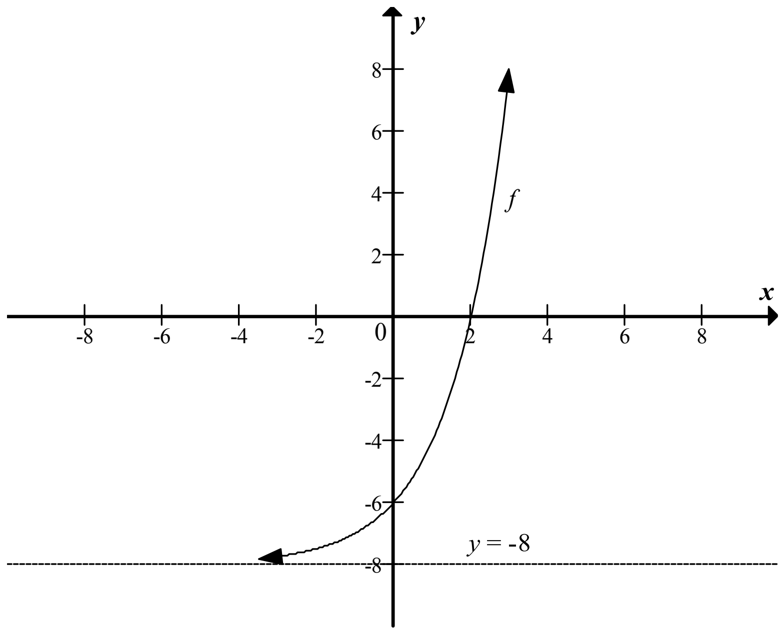
	$S_\infty - S_n = T_{n+1} + T_{n+2} + T_{n+3} + \ldots$ $= 10\left(\frac{1}{2}\right)^n\left[1 + \frac{1}{2} + \frac{1}{4} + \ldots\right]$ $= 10\left(\frac{1}{2}\right)^n\left[\frac{1}{1-\frac{1}{2}}\right]$ $= 20\left(\frac{1}{2}\right)^n$	✓ $10\left(\frac{1}{2}\right)^n\left[1 + \frac{1}{2} + \frac{1}{4} + \ldots\right]$ ✓ $\frac{1}{1-\frac{1}{2}}$ ✓answer (4)
	OR/*OF* $S_\infty - S_n = \frac{a}{1-r} - \frac{a(1-r^n)}{1-r}$ $= \frac{a - a + ar^n}{1-r}$ $= \frac{ar^n}{1-r}$ $= \frac{10\left(\frac{1}{2}\right)^n}{\frac{1}{2}}$ $= 20\left(\frac{1}{2}\right)^n$	✓ $\frac{a - a + ar^n}{1-r}$ ✓ $\frac{ar^n}{1-r}$ ✓ $\frac{10\left(\frac{1}{2}\right)^n}{\frac{1}{2}}$ ✓answer (4) **[10]**

QUESTION/*VRAAG* 3

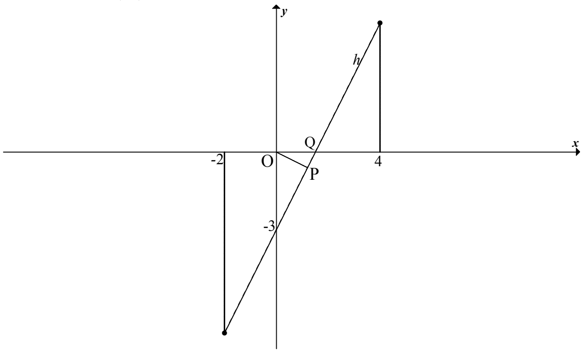
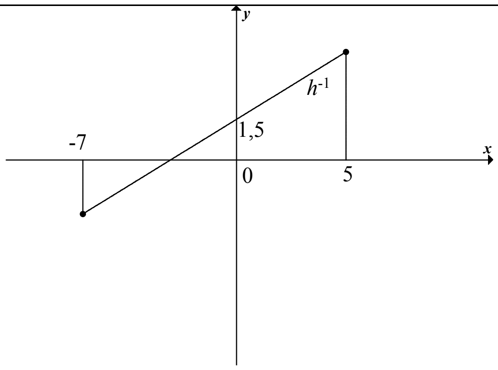
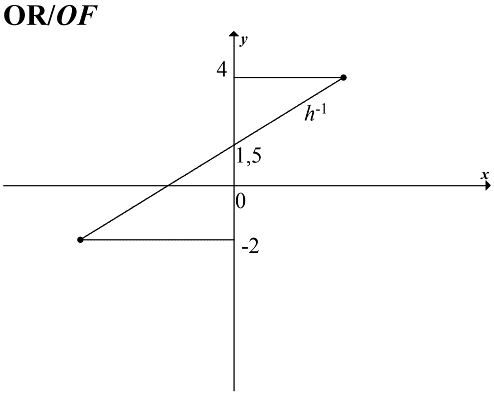
3.1	$d = 8$ $T_k = a + (k-1)d$ $= -3 + (k-1)(8)$ $= -3 + 8k - 8$ $= 8k - 11$	✓ d value ✓answer (2)
3.2	$\sum_{k=1}^{n}(8k-11)$ **OR/*OF*** $\sum_{k=0}^{n-1}(8(k+1)-11) = \sum_{k=0}^{n-1}(8k-3)$	✓for general term ✓lower and upper values in sigma notation (2)
3.3	$S_n = \frac{n}{2}[2a + (n-1)d]$ $= \frac{n}{2}[2(-3) + (n-1)(8)]$ $= \frac{n}{2}[-6 + 8n - 8]$ $= \frac{n}{2}[8n - 14]$ $= n(4n-7)$ $= 4n^2 - 7n$ **OR/*OF*** $S_n = \frac{n}{2}[2a + (n-1)d]$ $= \frac{n}{2}[2(-3) + (n-1)(8)]$ $= \frac{n}{2}[-6 + 8n - 8]$ $= \frac{n}{2}[8n - 14]$ $= 4n^2 - 7n$ **OR/*OF*** $S_n = \frac{n}{2}[a + l]$ $= \frac{n}{2}[-3 + 8n - 11]$ $= \frac{n}{2}[8n - 14]$ $= 4n^2 - 7n$	✓formula ✓substitution ✓ $\frac{n}{2}[8n-14]$ (3) ✓formula ✓substitution ✓ $\frac{n}{2}[8n-14]$ (3) ✓formula ✓substitution ✓ $\frac{n}{2}[8n-14]$ (3)

	OR/*OF* S_1 S_2 S_3 S_4 -3 2 15 36 5 13 21 8 8 $S_n = an^2 + bn + c$ $a = \frac{8}{2}$ $a = 4$ $S_1 = 4 + b + c = -3$ $b + c = -7$ ....(1) $S_2 = 16 + 2b + c = 2$ $2b + c = -14$.............(2) $b = -7$................(2)−(1) $c = 0$ Hence $S_n = 4n^2 - 7n$	$S_2 = -3 + 5 = 2$ $S_3 = 2 + 13 = 15$ $S_4 = 15 + 21$ ✓calculates S_1, S_2, S_3 and S_4, ✓ $a = 4$ ✓solves simultaneously for *b* and *c*. (3)
3.4.1	$Q_6 = -6 - 3 + 5 + 13 + 21 + 29$	✓✓ answer (2)
3.4.2	$Q_{129} = -6 + S_{128}$ $= -6 + 4(128)^2 - 7(128)$ $= 64634$ **OR/*OF*** Q_1 Q_2 Q_3 Q_4 -6 -9 -4 9 -3 5 13 8 8 8 $Q_n = an^2 + bn + c$ $a = 4$ $Q_1 = 4 + b + c = -6$ $b + c = -10$ ....(1) $Q_2 = 16 + 2b + c = -9$ $2b + c = -25$..............(2) $b = -15$................(2)−(1) $c = 5$ Hence $Q_n = 4n^2 - 15n + 5$ $Q_{129} = 4(129)^2 - 15(129) + 5$ $= 64\ 634$	✓✓ $-6 + 4(128)^2 - 7(128)$ ✓answer (3) ✓$a = 4$ ✓ $Q_n = 4n^2 - 15n + 5$ ✓answer (3) **[12]**

QUESTION/*VRAAG* 4

Given: $f(x)=2^{x+1}-8$		
4.1	$y=-8$	✓ $y=-8$ (1)
4.2		✓x-intercept ✓y-intercept ✓shape ✓asymptote (only if the graph does not cut the asymptote) (4)
4.3	$g(x)=2^{-x+1}-8$ **OR/*OF*** $g(x)=\left(\dfrac{1}{2}\right)^{x-1}-8$	✓answer (1) ✓answer (1) **[6]**

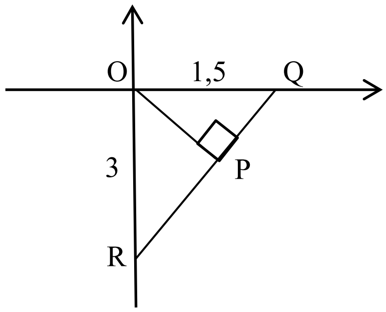
QUESTION/*VRAAG* 5

Given $h(x) = 2x - 3$ for $-2 \le x \le 4$.  		
5.1	For x-intercepts, $y = 0$ $2x - 3 = 0$ $x = 1{,}5$ $\text{Q}(1{,}5\ ;\ 0)$	✓ $x = 1{,}5$ ✓ $y = 0$ (2)
5.2	h: $x = -2: \quad y = 2(-2) - 3 = -7$ $x = 4: \quad y = 2(4) - 3 = 5$ Domain of $h^{-1}: -7 \le x \le 5$ **OR/*OF*** $[-7;\ 5]$	✓ $h(-2) = -7$ ✓ $h(4) = 5$ ✓ $-7 \le x \le 5$ (3)
5.3	 **OR/*OF***  	✓ y-intercept on a straight line ✓ line segment ✓ accurate endpoints (x or y or both) (3)

5.4	$h(x)=2x-3$ For the inverse of h, $x=2y-3$ $y=\dfrac{x+3}{2}$ $h^{-1}(x)=\dfrac{x+3}{2}$ $h(x)=h^{-1}(x)$ $2x-3=\dfrac{x+3}{2}$ $4x-6=x+3$ $x=3$ **OR/*OF*** $h(x)=2x-3$ h and h^{-1} intersect when $y=x$ $h(x)=x$ $2x-3=x$ $x=3$ **OR/*OF*** $h(x)=2x-3$ For the inverse of h, $x=2y-3$ $y=\dfrac{x+3}{2}$ $h^{-1}(x)=x$ $\dfrac{x+3}{2}=x$ $x+3=2x$ $x=3$	✓ $y=\dfrac{x+3}{2}$ ✓ $2x-3=\dfrac{x+3}{2}$ ✓ $x=3$ (3) ✓ $h(x)=x$ ✓ $2x-3=x$ ✓ $x=3$ (3) ✓ $y=\dfrac{x+3}{2}$ ✓ $\dfrac{x+3}{2}=x$ ✓ $x=3$ (3)

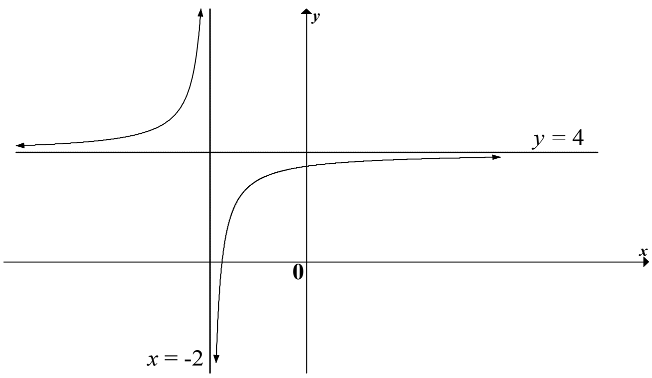
5.5	$\text{OP}^2 = (x-0)^2 + (y-0)^2$ $= x^2 + (2x-3)^2$ $= x^2 + 4x^2 - 12x + 9$ $= 5x^2 - 12x + 9$ For OP to be at its minimum, OP^2 has to be a minimum *Vir OP om minimum te wees, moet OP^2 'n minimum wees* $\frac{d(\text{OP}^2)}{dx} = 0$ **OR** / *OF* $x = -\frac{b}{2a}$ $10x - 12 = 0$ $= -\frac{-12}{2(5)}$ $\therefore x = \frac{6}{5}$ Minimum length of $\text{OP} = \sqrt{5\left(\frac{6}{5}\right)^2 - 12\left(\frac{6}{5}\right) + 9} = \sqrt{\frac{9}{5}}$ or $\frac{3}{\sqrt{5}}$ or 1,34 units	✓ $\text{OP}^2 = x^2 + y^2$ ✓substitute $y = 2x - 3$ ✓ $5x^2 - 12x + 9$ ✓x-value ✓answer (5)
	OR/*OF* For minimum distance OP ⊥ the line $m_h = 2$ (given) $m_{\text{OP}} = \frac{-1}{2}$ $\therefore$ OP has equation $y = \frac{-1}{2}x$ $\frac{-1}{2}x = 2x - 3$ $-x = 4x - 6$ $5x = 6$ $x_P = 1{,}2$ $y_P = -\frac{1}{2}(1{,}2) = -0{,}6$ $\text{OP} = \sqrt{(1{,}2-0)^2 + (-0{,}6-0)^2}$ $= 1{,}34$ or $\sqrt{1{,}8}$ units	✓ $m_{\text{OP}} = \frac{-1}{2}$ ✓equation of OP ✓ $\frac{-1}{2}x = 2x - 3$ ✓x-value ✓answer (5)

	OR/*OF*	
	For minimum distance OP $\perp$ the line	
	$O(0;0) \quad P(x;\, 2x-3) \quad Q\left(\frac{3}{2};0\right)$	✓ $OP^2 = x^2 + y^2$ ✓substitute $y = 2x-3$
	$OP^2 + PQ^2 = OQ^2$ (pythag)	
	$(x-0)^2 + (2x-3-0)^2 + \left(x-\frac{3}{2}\right)^2 + (2x-3-0)^2 = \left(\frac{3}{2}\right)^2$	✓ $10x^2 - 27x + 18$
	$x^2 + 4x^2 - 12x + 9 + x^2 - 3x + \frac{9}{4} + 4x^2 - 12x + 9 = \frac{9}{4}$	
	$10x^2 - 27x + 18 = 0$	
	$(5x-6)(2x-3) = 0$	✓x-value
	$x = \frac{6}{5}$ *or* $\frac{3}{2}$	
	Hence, $x = \frac{6}{5}$ at P	
	$OP^2 = x^2 + (2x-3)^2$	
	$= \left(\frac{6}{5}\right)^2 + \left(2\left(\frac{6}{5}\right) - 3\right)^2$	✓answer
	$= \frac{36}{25} + \frac{9}{25}$	(5)
	$= \frac{9}{5}$	
	$OP = 1{,}34$	
	OR/*OF*	
	For minimum distance OP $\perp$ the line	
	$\tan \hat{Q} = 2$	✓ $\tan \hat{Q} = 2$
	$\hat{Q} = 63{,}43^\circ$	✓ $\hat{Q} = 63{,}43^\circ$
	$\sin 63{,}43^\circ = \frac{OP}{1{,}5}$	✓ $\sin 63{,}43^\circ$ ✓ $\frac{OP}{1{,}5}$
	$OP = 1{,}34$	✓ answer (5)

	OR/*OF*	
	$OP=\sqrt{(x-0)^2+(y-0)^2}$	✓ $OP=\sqrt{(x-0)^2+(y-0)^2}$
	$=\sqrt{(x-0)^2+(2x-3-0)^2}$	✓substitute $y=2x-3$
	$=\sqrt{x^2+4x^2-12x+9}$	
	$=\sqrt{5x^2-12x+9}$	✓ $5x^2-12x+9$
	By using the chain rule (which is not in the CAPS):	
	$\frac{dOP}{dx}=\frac{1}{2}\left(5x^2-12x+9\right)^{\frac{-1}{2}}\cdot(10x-12)$	
	$0=\frac{1}{2}\left(5x^2-12x+9\right)^{\frac{-1}{2}}\cdot(10x-12)$	
	$0=\frac{1}{2}(10x-12)$	
	$0=5x-6$	
	$x=\frac{6}{5}$	✓x-value
	$OP=\sqrt{5\left(\frac{5}{6}\right)^2-12\left(\frac{6}{5}\right)+9}$	
	$=1{,}34$	✓answer (5)
	OR/*OF*	
	For minimum distance OP $\perp$ the line Let the y-intercept be R OR = 3 units OQ = $\frac{3}{2}$ units	
		
	RQ = $\frac{3}{2}\sqrt{5}$ (Pythagoras)	✓ RQ = $\frac{3}{2}\sqrt{5}$
	Area OQR = $\frac{1}{2}\times$ base $\times$ $\perp$height	
	$\frac{1}{2}.\text{OR}.\text{OQ}=\frac{1}{2}.\left(\frac{3}{2}\sqrt{5}\right).\text{OP}$	✓ $\frac{1}{2}.\left(\frac{3}{2}\sqrt{5}\right).\text{OP}$
	$\frac{1}{2}.3.\left(\frac{3}{2}\right)=\frac{1}{2}.\left(\frac{3}{2}\sqrt{5}\right).\text{OP}$	✓ $\frac{1}{2}.3.\left(\frac{3}{2}\right)$ ✓ equating
	$\text{OP}=\frac{3}{\sqrt{5}}=1{,}34$	✓ answer (5)

5.6.1	$f'(x) = 2x - 3$ Turning point at $x = \frac{3}{2}$ $f''(x) = 2 > 0$ or $f''\left(\frac{3}{2}\right) > 0$ f has a local minimum at $x = \frac{3}{2}$ *f het 'n lokale minimum by $x = \frac{3}{2}$* **OR/*OF*** $h(x) = f'(x) < 0$ for $x \in (-2\ ;\ 1{,}5) \Rightarrow f$ is decreasing on the left of Q / *f is dalend links van Q.* $h(x) = f'(x) > 0$ for $x \in (1{,}5\ ;\ 4) \Rightarrow f$ is increasing on the right of Q / *f is stygend regs van Q.* $\therefore f(x)$ has a local minimum when $x = \frac{3}{2}$ / *$\therefore f(x)$ het 'n lokael minimum by $x = \frac{3}{2}$* **OR/*OF*** $f(x) = x^2 - 3x + c$ f has a minimum value since $a > 0$ *f het 'n minimum waarde omdat $a > 0$*	✓ Turning point at $x = \frac{3}{2}$ ✓ $f''(x) = 2 > 0$ (2) ✓decreasing left of Q ✓increasing right of Q (2) ✓ $f(x) = x^2 - 3x + c$ ✓ explanation (2)
5.6.2	$m = f'(4) = h(4) = 5$	✓answer (1) **[19]**

QUESTION/*VRAAG* 6

6.1.1	$T(0;18)$	✓ $(0;18)$ (1)
6.1.2	$-2x^2+18=0$ $(x-3)(x+3)=0$ Q(3 ; 0) **OR/*OF*** $-2x^2+18=0$ $x^2=9$ Q(3 ; 0)	✓$y=0$ ✓factors ✓$x=3$ (3) ✓ $y=0$ ✓ $x^2=9$ ✓ $x=3$ (3)
6.1.3	x-coordinate of S is 4,5/*x-koördinaat van S is 4,5* By symmetry about the line x = 4,5/*Deur simmetrie om die lyn x = 4,5:* R = (4,5 + 4,5–3 ; 0) = (6 ; 0)	✓ $x=6$ ✓ $y=0$ (2)
6.1.4	For all $x \in \mathbf{R}$ **OR/*OF*** $(-\infty;\infty)$	✓✓answer (2)
6.2	If C$(x;y)$is the centre of the hyperbola/*As C*$(x;y)$*die middelpunt is van die hiperbool* $y=x+6$ and $x=-2$ $\therefore y=-2+6=4$ 	✓✓ asymptote $y=4$ ✓asymptote $x=-2$ ✓shape (increasing hyperbolic function) (4) **[12]**

QUESTION/*VRAAG* 7

7.1	R450 000	✓answer (1)
7.2	$A = P(1-i)^n$ $f(x) = 450000(1-i)^x$ $243\ 736{,}90 = 450000(1-i)^4$ $i = 1 - \sqrt[4]{\dfrac{243\ 736{,}90}{450000}}$ $i = 0{,}1421$ The rate of depreciation is 14,21 % p.a. *Die waardeverminderingskoers is 14,21% p.j.*	✓substitution of 450 000 into correct formula ✓substitution of $(4;\ 243\ 736{,}90)$ into correct formula ✓making *i* the subject ✓answer (4)
7.3	At T : $A = P(1+i)^n$ $g(x) = 450000(1+i)^x$ $a = 450000(1+0{,}081)^4$ $= R614490{,}66$	✓i = 0,081 & n = 4 ✓correct substitution into formula ✓answer (3)
7.4	$\text{Future Value} = R614\ 490{,}66 - R243\ 736{,}90$ $= R370\ 753{,}76$ Let x be the value of monthly payment $F_v = \dfrac{x\left[(1+i)^n - 1\right]}{i}$ $370753{,}76 = \dfrac{x\left[\left(1+\dfrac{0.062}{12}\right)^{36} - 1\right]}{\dfrac{0.062}{12}}$ $x = R9397{,}11$	✓R370 753,76 ✓$i = \dfrac{0{,}062}{12}$ ✓n = 36 ✓substitution into correct formula ✓answer (5) **[13]**

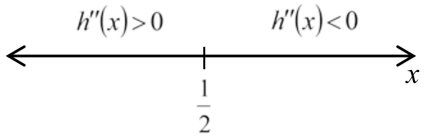
QUESTION/*VRAAG* 8

8.1	$f(x+h) = (x+h)^2 - 3(x+h)$ $= x^2 + 2xh + h^2 - 3x - 3h$ $f(x+h) - f(x) = x^2 + 2xh + h^2 - 3x - 3h - (x^2 - 3x)$ $= 2xh + h^2 - 3h$ $f'(x) = \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $= \lim\limits_{h\to 0}\dfrac{2xh + h^2 - 3h}{h}$ $= \lim\limits_{h\to 0}\dfrac{h(2x+h-3)}{h}$ $= \lim\limits_{h\to 0}(2x+h-3)$ $= 2x - 3$	✓finding $f(x+h)$ ✓ $2xh+h^2-3h$ ✓formula ✓factorisation ✓answer (5)
	OR/*OF* $f'(x) = \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $= \lim\limits_{h\to 0}\dfrac{(x+h)^2 - 3(x+h) - (x^2 - 3x)}{h}$ $= \lim\limits_{h\to 0}\dfrac{x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x}{h}$ $= \lim\limits_{h\to 0}\dfrac{2xh + h^2 - 3h}{h}$ $= \lim\limits_{h\to 0}\dfrac{h(2x+h-3)}{h}$ $= \lim\limits_{h\to 0}(2x+h-3)$ $= 2x - 3$	✓formula ✓finding $f(x+h)$ ✓ $2xh+h^2-3h$ ✓factorisation ✓answer (5)
8.2.1	$y = \left(x^2 - \dfrac{1}{x^2}\right)^2$ $y = x^4 - 2 + \dfrac{1}{x^4}$ $= x^4 - 2 + x^{-4}$ $\dfrac{dy}{dx} = 4x^3 - 4x^{-5}$ **OR/*OF***	✓ $x^4 - 2 + \dfrac{1}{x^4}$ ✓ $4x^3$ ✓ $-4x^{-5}$ (3)

	By using the chain rule (which is not part of CAPS): $y=\left(x^2-x^{-2}\right)^2$ $\frac{dy}{dx}=2\left(x^2-x^{-2}\right)\left(2x+2x^{-3}\right)$ $=2\left(2x^3+2x^{-1}-2x^{-1}-2x^{-5}\right)$ $=2\left(2x^3-2x^{-5}\right)$ $=4x^3-4x^{-5}$	✓✓✓ $2\left(x^2-x^{-2}\right)\left(2x+2x^{-3}\right)$ (3)
8.2.2	$D_x\left[\frac{(x-1)(x^2+x+1)}{x-1}\right]$ $=D_x\left[x^2+x+1\right]$ $=2x+1$ **OR/*OF*** By using the quotient rule (with is not part of CAPS): $D_x\left[\frac{x^3-1}{x-1}\right]$ $=\frac{3x^2(x-1)-\left(x^3-1\right)}{(x-1)^2}$	✓factorisation ✓ x^2+x+1 ✓ $2x+1$ (3) ✓✓✓ $\frac{3x^2(x-1)-\left(x^3-1\right)}{(x-1)^2}$ (3) **[11]**

QUESTION/*VRAAG* 9

9.1	Substitute Q(2; 10) into $h(x) = -x^3 + ax^2 + bx$ $-2^3 + a(2^2) + b(2) = 10$ $-8 + 4a + 2b = 10$ $2a + b = 9$ ....line 1 $h'(x) = -3x^2 + 2ax + b$ At Q : $h'(2) = 0$ $-3(2)^2 + 2a(2) + b = 0$ $-12 + 4a + b = 0$ $4a + b = 12$ ....line 2 line 2 – line 1 : $2a = 3$ $a = \frac{3}{2}$ Substitute in line 1 : $b = 6$	✓substitute Q into h ✓finding derivative ✓ $h'(2)$ ✓equating derivative to 0 ✓solving simultaneously for a and b (5)
9.2	$f(-1) = -(-1)^3 + \frac{3}{2}(-1)^2 + 6(-1)$ $= -3{,}5$ Average gradient/*Gemiddelde gradiënt* $= \frac{f(x_Q) - f(x_P)}{x_Q - x_P}$ Average gradient/ *Gemiddelde gradiënt* $= \frac{10 - (-3{,}5)}{2 - (-1)}$ $= 4{,}5$	✓ $f(-1) = -3{,}5$ ✓formula ✓substitution ✓answer (4)

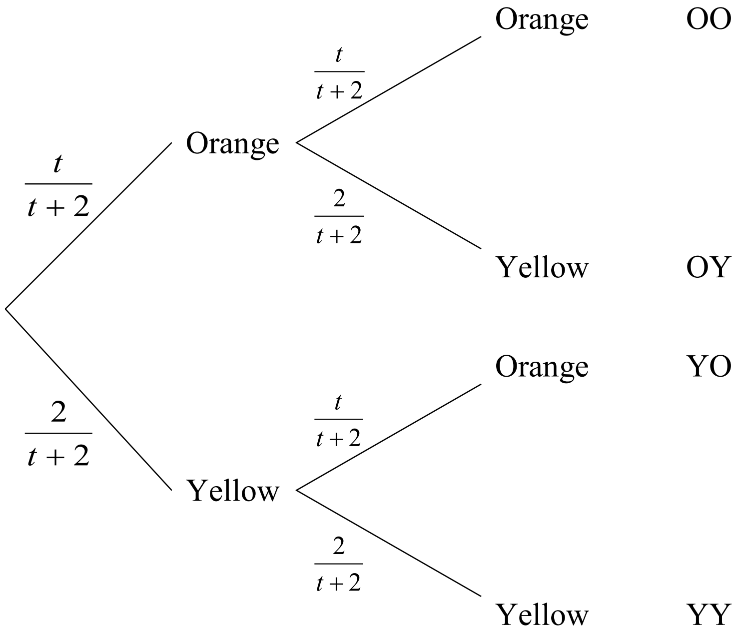
9.3	$h'(x) = -3x^2 + 3x + 6$ $h''(x) = -6x + 3$ $= -3(2x - 1)$  For $x < \frac{1}{2}$, h is concave up and for $x > \frac{1}{2}$, h is concave down *Vir* $x < \frac{1}{2}$*, is h konkaaf na bo en vir* $x > \frac{1}{2}$*, is h konkaaf na onder* $\therefore$ concavity changes at $x = \frac{1}{2}$ / $\therefore$ *konkwiteit verander by* $x = \frac{1}{2}$	✓ $h'(x) = -3x^2 + 3x + 6$ ✓ $h''(x) = -6x + 3$ ✓explanation using $h''(x)$ (3)
9.4	The graph of h has a point of inflection at $x = \frac{1}{2}$ / *Die grafiek van h het 'n buigpunt by* $x = \frac{1}{2}$. **OR/*OF*** The graph of h changes from concave up to concave down at $x = \frac{1}{2}$ / *Die grafiek van h verander by* $x = \frac{1}{2}$ *van konkaaf op na konkaaf af*	✓answer (1) ✓answer (1)
9.5	Gradient of g is -12/*Gradiënt van g is –12* Gradient of tangent is/*Gradiënt van die raaklyn is*: $h'(x) = -3x^2 + 3x + 6$ $h'(x) = -12$ $-3x^2 + 3x + 6 = -12$ $3x^2 - 3x + 18 = 0$ $x^2 - x + 6 = 0$ $(x - 3)(x + 2) = 0$ $x = -2$ only	✓ $h'(x) = -3x^2 + 3x + 6$ ✓ $h'(x) = -12$ ✓factors ✓selection of x-value (4) **[17]**

QUESTION/*VRAAG* 10

10.1	$\frac{h}{r}=\tan 60^0$ $r=\frac{h}{\tan 60^0}$ $\therefore r=\frac{h}{\sqrt{3}}$	✓ $\frac{h}{r}=\tan 60^0$ ✓answer (2)
10.2	$V_{cone}=\frac{1}{3}\pi r^2 h$ $=\frac{1}{3}\pi\left(\frac{h}{\sqrt{3}}\right)^2 h$ $=\frac{1}{9}\pi h^3$ $\frac{dV}{dh}=\frac{1}{3}\pi h^2$ $\left.\frac{dV}{dh}\right\|_{h=9}=\frac{1}{3}\pi(9)^2$ $=27\pi$ or $84{,}82\ \text{cm}^3/\text{cm}$	✓formula ✓substitution of the value of r in terms of h ✓simplified volume answer ✓derivative ✓answer (5) **[7]**

QUESTION/*VRAAG* 11

11.1	$P(A) \times P(B)$ $= 0{,}2 \times 0{,}63$ $= 0{,}126$ i.e. $P(A) \times P(B) = P(A \text{ and } B)$ Therefore A and B are independent/*Dus is A en B onafhanklik*	✓$0{,}2 \times 0{,}63$ ✓$P(A) \times P(B)=P(A$ and B) ✓conclusion (3)
11.2.1	$7^7 = 823\ 543$	✓✓7^7 (2)
11.2.2	$7! = 5040$	✓✓7! (2)
11.2.3	There are 3 vowels $\Rightarrow$ 3 options for first position There are 4 consonants $\Rightarrow$ 4 options for last position The remaining 5 letters can be arranged in $5\times4\times3\times2\times1$ ways $3\times(5\times4\times3\times2\times1)\times4 = 1440$ *Daar is 3 klinkers* $\Rightarrow$ *3 opsies vir die eerste posisie* *Daar is 4 konsonante* $\Rightarrow$ *4 opsies vir die laaste posisie* *Die oorblywende 5 letters kan as volg gerangskik word* $5\times4\times3\times2\times1$ ways/*maniere* $3\times(5\times4\times3\times2\times1)\times4 = 1440$	✓$\times3$ ✓$\times4$ ✓$5\times4\times3\times2\times1$ ✓answer (4)

11.3		✓$P(O) = \left(\frac{t}{t+2}\right)$ ✓$P(Y) = \left(\frac{2}{t+2}\right)$
	$\text{P(Orange, Orange)} + \text{P(Yellow, Yellow)} = \frac{52}{100}$ $\left(\frac{t}{t+2}\right)\left(\frac{t}{t+2}\right)+\left(\frac{2}{t+2}\right)\left(\frac{2}{t+2}\right)=\frac{52}{100}$ $\frac{t^2}{t^2+4t+4}+\frac{4}{t^2+4t+4}=\frac{13}{25}$ $25(t^2+4)=13(t^2+4t+4)$ $3t^2-13t+12=0$ $(3t-4)(t-3)=0$ $t=3$ There are 3 orange balls in the bag/*Daar is 3 oranje balle in die sak*	✓$P(O,O) = \left(\frac{t}{t+2}\right)^2$ ✓ $P(Y,Y) = \left(\frac{2}{t+2}\right)^2$ ✓ $\left(\frac{t}{t+2}\right)\left(\frac{t}{t+2}\right)+\left(\frac{2}{t+2}\right)\left(\frac{2}{t+2}\right)=\frac{52}{100}$ ✓ $t=3$ (no ca) (6) **[17]**
	TOTAL/TOTAAL:	**150 marks**